Reasoning First
Teaching Mathematical Reasoning in Year 2

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Introduction

Two types of ability are essential for children to succeed in mathematics in school: they must have a good grasp of numbers and relations between numbers, and they must be able to reason logically about quantities. The first of these abilities is **number sense**; it is essential for learning arithmetic. The second is **quantitative reasoning**; it is essential for problem solving. These two abilities are related, but they need to be taught differently. Our previous research (Nunes, Bryant, Barros, & Sylva, 2011) showed that each of these two abilities, assessed when the children were aged 8-9 years, predicted how well they performed in KS2 and KS3 mathematics, and that quantitative reasoning was the stronger predictor.

Our research (Nunes, Bryant, Evans, Bell, Gardner, Gardner, & Carraher, 2007) has also shown that an effective way to help children who are at risk for having difficulty in learning mathematics is to improve their quantitative reasoning ability. Children in Year 1 who did not do well in a mathematical assessment when they started school made large gains by participating in our quantitative reasoning programme delivered in small groups. At the end of Year 2 they performed just above the 50th percentile (average level) for the UK in KS1; in contrast, a comparable group that did not participate in the reasoning programme performed at the 28th percentile.

This programme for teaching mathematical reasoning in Year 2 was developed for whole class and includes activities to promote both **number sense** and **quantitative reasoning**. It was assessed in a randomized controlled trial funded by the Education Endowment Foundation (EEF) in a project that involved 55 schools and 2,087children. The external evaluators chose the Progress in Maths test to evaluate the impact of the programme. They found that the children who participated in the Mathematical Reasoning programme made an extra three months of progress in mathematics than those who did not participate.

The programme is organized in 12 units to be taught once a week in the time scheduled for mathematics teaching; no extra time is required. Each lesson starts with a whole class introduction to the concept. The teacher uses a PowerPoint presentation, often with the support of manipulatives, to explain the concept. The children should be active all the time, answering questions and explaining how they arrived at their answers. They are encouraged to use actions and manipulatives to explain their reasoning. During the second part of the lesson, pupils are divided in two groups. Some children, supervised by a TA, play computer games that give them further practice with the concept. The others work with the teacher to do further work, extend the concept, or for pre-teaching of the next concept.

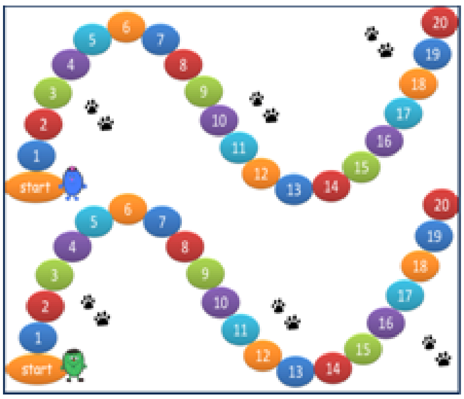


Figure 1: The caterpillar game used to promote counting on.

Teaching Mathematical Reasoning in Year 2

Pre-teaching gives children who might have difficulty with the concept that will be taught in the next lesson the chance to start thinking about it before the lesson. This gives children who often struggle the experience of success and greater enjoyment later, when the concept is taught to the whole class. Children who find mathematics easier have the chance to be exposed to more mathematical ideas through extension games and exercises. All children play computer games that help them achieve greater facility and flexibility with numbers and problem solving. The games are available on the programme website (www.numeracygames.org.uk).

The **number sense** and **quantitative reasoning** activities are expected to develop in tandem: as the children can think about larger numbers, the problems that they are asked to solve involve larger quantities. This means that the two strands must be intertwined during teaching because they reinforce each other.

The next pages offer a taste of the simpler activities. The activities become more difficult throughout the programme. There is no need to repeat lessons when the children find them hard; there will be other activities about the same concepts to give them further learning opportunities.

The number sense activities

The number sense strand focuses on two key ideas: (**1) additive composition and place value representation and (2) the inverse relation between addition and subtraction**.

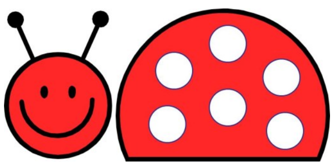
Additive composition

Additive composition refers to the fact that any number can be thought of as the sum of two other numbers. This means that children should not think of numbers simply as words in a sequence, like words in a poem, but should learn to think of the relations that exist between them. Children cannot really understand what five is unless they also know that it is the same as 4+1 or 3+2 or 6-1. These activities start with the caterpillar game, in which children are asked to **count on** from a number on the board. Counting on is a basic skill for learning about additive composition. Figure 1 on the previous page shows the caterpillar game.

This component is at the heart of understanding any number system with a base and of representing numbers in writing using the place-value system: 127, for example, means 100+20+7. Research (Nunes & Bryant, 1996) has shown that many children who can count out, for example, 17p using 1p coins cannot count out 17p using one 10p and seven 1p coins. These children have learned to count but have not grasped additive composition. The activities that promote understanding of additive composition involve equivalence and decomposition (see Figure 2)

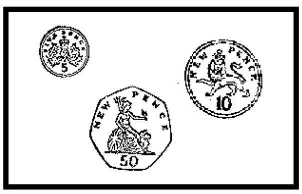
Unit 1

In this activity, the children are told that they need to fill the spots with 2p coins. They see a list of coins on a page, some 1p and some 2p, and must say how many spots they can fill by exchanging two 1p coins for one 2p coin.



Unit 3

How much money do you have? Change the money into 10p and 1p coins and fill in the table.



tens	ones

Figure 2: In Unit 1 the children learn about equivalence in action as they exchange two 1p coins for one 2p coin. After learning to compose the same value in different ways, children learn in Unit 3 to decompose values only in tens and ones. This prepares them to write numbers using place value.

Children's understanding of additive composition develops over time. At first they understand it with small numbers; later they can use the same idea with larger numbers, including tens and hundreds, and later including thousands. This understanding forms the basis for them to master writing numbers using place value. Children need to practise the idea of additive composition with larger numbers in order to understand place value well.

Two games in our programme extend the children's understanding of additive composition: the *Gremlins Game* and *Seven and a Half.*

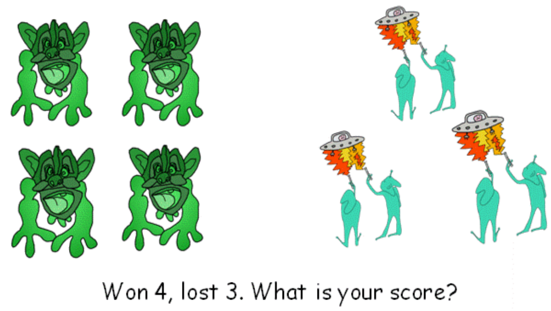


Figure 3: The Gremlins Game.

In the Gremlins Game, you win a point if you hit a Gremlin and you lose a point if they hit your spaceship. In Figure 3, the children learn to think of 1 as the same as 4-3, not just as the first number in the counting sequence.

The inverse relation between addition and subtraction

This is a topic that deserves special attention because it connects both to number understanding and to quantitative reasoning. On the whole, pre-school children understand that if you add, for example, blocks to a row of blocks, you will have more. They also understand that if you take away some blocks from a row of blocks, you will have fewer. But many do not understand that if you add and take away the same number of blocks, you have just as many as you had before. In other words, they do not understand well the idea that a number remains the same if these two transformations cancel each other out.

The teaching activities designed to promote children's thinking about this inverse relation start with a demonstration that is often quite clear to them: we show them a row of blocks of one colour - for example, red - and let them count the blocks. We then cover the middle of the row so that they can no longer count the blocks. Then we add blocks of a different colour - say, black - to the row and take away the same black blocks. We ask them to say, without counting, how many blocks are now in the row. Because the blocks are of a different colour, it is obvious to the children that the red row has not changed. This easy start gives them a grounding for reasoning about how the number of blocks changes: when we add 5 black blocks and take 4 away, they see that 1 was left in the row. When we add 2 and take away 3, they see that we took 1 red away. Through a carefully planned series of manoeuvres illustrated in Figure 4, we help them think about the inverse relation between addition and subtraction, and they make good progress both in understanding more about numbers and about quantitative reasoning.

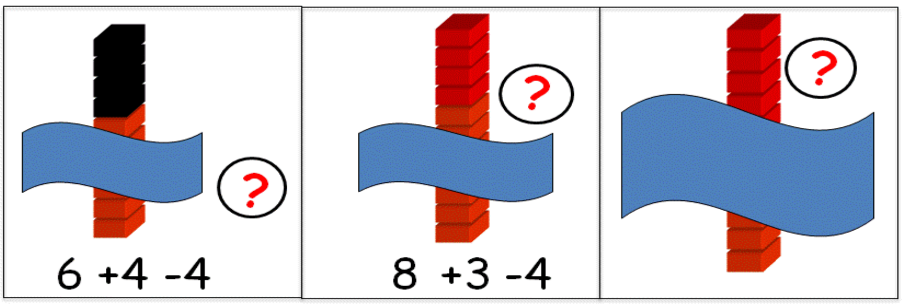


Figure 4: In the easier items of the games about the inverse relation between addition and subtraction, the information is made salient by the use of colours; the numbers are all visible when the question is posed. In the more difficult items, the blocks are all of the same colour and the information is less salient. In later items, the information has to be recalled; it appears on the screen and then disappears before the question comes up.

The quantitative reasoning activities

There are two types of quantitative reasoning activities. One type is about **part-whole or additive reasoning** (which covers addition and subtraction) and the other is about **one-to-many correspondence or multiplicative reasoning** (which includes multiplication and division).

Part-whole and additive reasoning

Part-whole relations form the core of reasoning about addition and subtraction: in short, additive reasoning. When children master the logic of part-whole relations, they can solve many problems that are difficult to start with. Most Year 2 children will find this problem easy: Holly had 5 books; Granny gave her 3 books; how many does she have now? However, many will find this problem difficult: Holly had some books; Granny gave her 3 books; now she has 8; how many did she have before? Clearly, both problems involve reasoning about part-whole relations. In both, the number of books now is the sum of the two parts - the number of books that Holly had and the number that Granny gave her. But the second problem involves some transformation of the information before calculation, i.e. the children know how many books Holly got from Granny and the total, but need to think of what the situation was like **before** she got the books from Granny. This is another use of reasoning about the inverse relation between addition and subtraction: before Holly got 3 books from Granny, she had 3 books less. Our additive reasoning activities were designed to encourage children to think of part-whole relations and the operations of addition and subtraction from different perspectives.

One-to-many correspondence and multiplicative reasoning

Many people are surprised to find that even children in Year 1 can successfully solve multiplication and division problems when they are offered the right sort of support. Look at the problem in Figure 5, which also shows how one child got to the answer. The reasoning is perfect and we should encourage children to develop it further. Teachers won't be surprised that Year 2 children can solve the problem in Figure 6. Both problems are solved by establishing one-to-many correspondence between two quantities and Year 2 children can do this when they have the right support—i.e. when they have manipulatives to represent both quantities, e.g. rabbits and biscuits.

Like additive reasoning, multiplicative reasoning is necessary for understanding relations between numbers and written numerical representation. When we write the number 25, the digit 2 represents 2x10. When we encourage children to think more about one-to-many correspondence, which they understand quite well, they can also make progress in understanding numbers written using a place-value system. Multiplicative reasoning is also the basis for the development of quantitative reasoning skills of great significance in problem solving throughout primary school: for example proportions problems.

Figure 5. 4 rabbits live in each of the 3 houses. How many carrots do we need to give one to each rabbit?

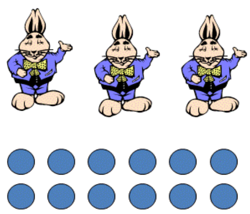


Figure 6. How many biscuits will each rabbit receive if we share the biscuits fairly?

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